

MATHEMATICS

PART II

Textbook for Class XII



12080



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Foreword

The National Curriculum Framework, 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

NCERT appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Professor P.K. Jain for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

New Delhi
20 November 2006

Director
National Council of Educational
Research and Training

Rationalisation of Content in the Textbooks

In view of the COVID-19 pandemic, it is imperative to reduce content load on students. The National Education Policy 2020, also emphasises reducing the content load and providing opportunities for experiential learning with creative mindset. In this background, the NCERT has undertaken the exercise to rationalise the textbooks across all classes. Learning Outcomes already developed by the NCERT across classes have been taken into consideration in this exercise.

Contents of the textbooks have been rationalised in view of the following:

- Overlapping with similar content included in other subject areas in the same class
- Similar content included in the lower or higher class in the same subject
- Difficulty level
- Content, which is easily accessible to students without much interventions from teachers and can be learned by children through self-learning or peer-learning
- Content, which is irrelevant in the present context

This present edition, is a reformatted version after carrying out the changes given above.

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Preface

The National Council of Educational Research and Training (NCERT) had constituted 21 Focus Groups on Teaching of various subjects related to School Education, to review the National Curriculum Framework for School Education - 2000 (NCFSE - 2000) in face of new emerging challenges and transformations occurring in the fields of content and pedagogy under the contexts of National and International spectrum of school education. These Focus Groups made general and specific comments in their respective areas. Consequently, based on these reports of Focus Groups, National Curriculum Framework (NCF)-2005 was developed.

NCERT designed the new syllabi and constituted Textbook Development Teams for Classes XI and XII to prepare textbooks in Mathematics under the new guidelines and new syllabi. The textbook for Class XI is already in use, which was brought in 2005.

The first draft of the present book (Class XII) was prepared by the team consisting of NCERT faculty, experts and practicing teachers. The draft was refined by the development team in different meetings. This draft of the book was exposed to a group of practicing teachers teaching Mathematics at higher secondary stage in different parts of the country, in a review workshop organised by the NCERT at Delhi. The teachers made useful comments and suggestions which were incorporated in the draft textbook. The draft textbook was finalised by an editorial board constituted out of the development team. Finally, the Advisory Group in Science and Mathematics and the Monitoring Committee constituted by the HRD Ministry, Government of India have approved the draft of the textbook.

In the fitness of things, let us cite some of the essential features dominating the textbook. These characteristics have reflections in almost all the chapters. The existing textbook contains thirteen main chapters and two appendices. Each chapter contains the followings :

- Introduction: Highlighting the importance of the topic; connection with earlier studied topics; brief mention about the new concepts to be discussed in the chapter.
- Organisation of chapter into sections comprising one or more concepts/ subconcepts.
- Motivating and introducing the concepts/subconcepts. Illustrations have been provided wherever possible.

- Proofs/problem solving involving deductive or inductive reasoning, multiplicity of approaches wherever possible have been inducted.
- Geometric viewing / visualisation of concepts have been emphasized whenever needed.
- Applications of mathematical concepts have also been integrated with allied subjects like Science and Social Sciences.
- Adequate and variety of examples/exercises have been given in each section.
- For refocusing and strengthening the understanding and skill of problem solving and applicabilities, miscellaneous types of examples/exercises have been provided involving two or more subconcepts at a time at the end of the chapter. The scope of challenging problems to talented minority have been reflected conducive to the recommendation as reflected in NCF-2005.
- For more motivational purpose, brief historical background of topics have been provided at the end of the chapter and at the beginning of each chapter, relevant quotation and photograph of eminent mathematician who have contributed significantly in the development of the topic undertaken, are also provided.
- Lastly, for direct recapitulation of main concepts, formulas and results, brief summary of the chapter has also been provided.

I am thankful to Professor Krishan Kumar, Director, NCERT who constituted the team and invited me to join this national endeavour for the improvement of Mathematics education. He has provided us with an enlightened perspective and a very conducive environment. This made the task of preparing the book much more enjoyable and rewarding. I express my gratitude to Professor J. V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics, for his specific suggestions and advice towards the improvement of the book from time to time. I, also, thank Professor G. Ravindra, Joint Director, NCERT for his help from time to time.

I express my sincere thanks to Professor Hukum Singh, Chief Coordinator and Head, DESM, Dr. V. P. Singh, Coordinator and Professor, S. K. Singh Gautam who have been helping for the success of this project academically as well as administratively. Also, I would like to place on records my appreciation and thanks to all the members of the team and the teachers who have been associated with this noble cause in one or the other form.

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THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a ¹**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the ²[unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

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